

# The Lateral Surface Phonon Modes in Bi-layer Dielectrics with Different Values of Permittivity

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## Abstract

Surface phonon polaritons were electromagnetic waves that propagated at the surface of dielectrics. These polaritons had many attentions since the frequency can reach in the order of terahertz. Hence, it can be employ in terahertz technology. One of the general method to study polaritons was by solving Maxwell's equations. The results were dispersion relations which showed the characteristic of the polaritons. In this study, we analysed polaritons of a bilayer dielectrics which was comprised of two dielectrics which had different permittivities. We set the thickness of each dielectric was smaller than the wavelength so that we could use effective medium method. In this approximation, we can treat a multilayer system as a single effective medium. We also assumed that the surface modes propagated across the thickness of each dielectric. It was found that when the resonant frequency of the two dielectrics were considerably wide, then we had two branches of surface polaritons. In this result, each branch was represented each dielectric. The interesting result appeared when there was intersection of the frequencies near resonant frequency of the dielectrics. In this condition, we found only one branch of surface modes.

**Keyword:** bilayer dielectrics, dispersion relation, phonon polaritons

## Introduction

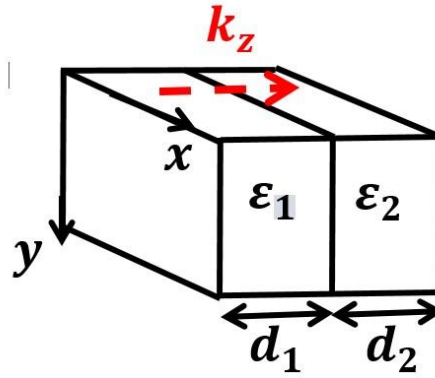
Phonon polaritons could be defined as electromagnetic waves that moved in the dielectrics or ferroelectrics. These propagated electromagnetic waves had modified by molecular vibration at the lattices of the dielectrics or ferroelectrics[1,2]. The dielectrics interacted to the electromagnetic waves through permittivity. Hence the optical frequencies of dielectrics were important variables in determined generated surface polaritons. Phonon polaritons had been studied a lot [3-5] because of their potential application in high speed signal and terahertz spectroscopy[6-8]. Phonon polariton had also studied in multilayer geometry[9-11], since it can modify the characteristic of the polaritons which was generated in single material.

In this paper, we wanted to study the characteristic of surface phonon polariton in bilayer system which was comprised of two dielectric materials with different optical resonant frequencies. Since the surface modes of phonon polaritons were determined by optical transversal and longitudinal frequencies[1], then the usage of bilayer structure would significantly modified the surface polariton

Here, we used the effective medium approximation[12] by setting the wavelength of electromagnetic waves bigger than the width of each dielectric layer. Using this method, we could consider the bilayer system as a single effective medium which had effective permittivity.

## The method and formulation

In this study, the semi-infinite geometry of lateral bilayer system was shown in Figure 1. Bilayer was comprised of dielectric 1 with permittivity  $\epsilon_1$  and dielectric 2 with permittivity  $\epsilon_2$ . Here, the parameter of dielectric 1 represented real dielectric InP. The parameters for dielectric 2 was set randomly. The interface between dielectric 1 and dielectric 2 was arranged at  $x$ - $y$  plane. The surface of the bilayer system was placed at  $x$ - $z$  plane. The width of layer 1 and 2 were represented by  $d_1$  and  $d_2$ . Here, the surface mode was assumed to propagate in  $z$  direction.



**Figure 1.** Geometry of the lateral bilayer. The interface was located at  $x$ - $y$  plane while the surface of bilayer structure was set at  $x$ - $z$  plane.

In this study, the surface phonon polariton was set in transverse magnetic (TM) modes where magnetic field was perpendicular to the plane of incident ( $H_x$ ). Hence the electric field components which were involved in this modes were  $E_y$  and  $E_z$ . The effective medium approximation of the bilayer system was derived by using the interface as a reference plane ( $x$ - $y$  plane). Employing the continuity of tangential electric field  $E_y$ , we had

$$E_1^y = E_2^y = \bar{E}_y. \quad (1)$$

The component of the electric displacement  $D_y$  field could be written as  $D_1^y = \varepsilon_1^y \bar{E}_y$  and  $D_2^y = \varepsilon_2^y \bar{E}_y$ . Then, the average of  $D_y$  field could be stated as

$$\bar{D}_y = \frac{d_1 D_1^y + d_2 D_2^y}{d_1 + d_2} = (f_1 \varepsilon_1^y + f_2 \varepsilon_2^y) \bar{E}_y. \quad (2)$$

Since  $\bar{D}_y = \varepsilon_{yy}^{eff} \bar{E}_y$ , then we could define the  $y$  component of effective permittivity as  $\varepsilon_{yy}^{eff} = f_1 \varepsilon_1^y + f_2 \varepsilon_2^y$ . (3)

Using similar procedure for the  $x$  component, we had

$$\varepsilon_{xx}^{eff} = f_1 \varepsilon_1^x + f_2 \varepsilon_2^x \quad (4)$$

where  $f$  was the fraction of the layer. Employing the continuity of normal  $\mathbf{D}$  field  $D_z$  refer to the interface plane and also the average of electric field  $E_z$ , we get

$$\frac{\varepsilon_{zz}^{eff}}{\varepsilon_1^z} = \left( \frac{f_1}{\varepsilon_1^z} + \frac{f_2}{\varepsilon_2^z} \right)^{-1}. \quad (5)$$

The above effective permittivity components were used in derivation of surface modes dispersion relation. Since the electromagnetic waves were in TM modes, we set the magnetic component as

$$\vec{H} = \hat{x} H_m e^{-\beta y} e^{i(k_z z - \omega t)} \quad \text{for } y > 0 \quad (6a)$$

$$\vec{H} = \hat{x} H_0 e^{\beta_0 y} e^{i(k_z z - \omega t)} \quad \text{for } y < 0 \quad (6b)$$

where  $\beta$  and  $\beta_0$  were represented as attenuation constant of bilayer system and vacuum. These attenuation constant can be obtained by solving Maxwell equations using Eqs.(6a) and (6b) which resulted as:  $\beta_0 = \left[ k_z^2 - \left( \frac{\omega}{c} \right)^2 \right]^{1/2}$  for vacuum, and

$$\beta = \left[ \left( \frac{\varepsilon_{zz}^{eff}}{\varepsilon_{yy}^{eff}} \right) k_z^2 - \varepsilon_{zz}^{eff} \left( \frac{\omega}{c} \right)^2 \right]^{1/2}. \quad (7)$$

for bilayer system.

The dispersion relation for the surface modes was obtained firstly by defining electric displacement field  $\vec{D}$ , magnetic induction field  $\vec{B}$  and magnetic field  $\vec{H}$ . Then, using continuity condition at the surface of the bilayer system ( $y = 0$ ) for tangential  $\vec{H}$ , tangential  $\vec{E}$ , normal  $\vec{B}$  and normal  $D$ , the relation between frequency  $\omega$  and propagation vector  $\vec{k}$  as

$$\varepsilon_{zz}^{eff} \beta_0 + \beta = 0. \quad (8)$$

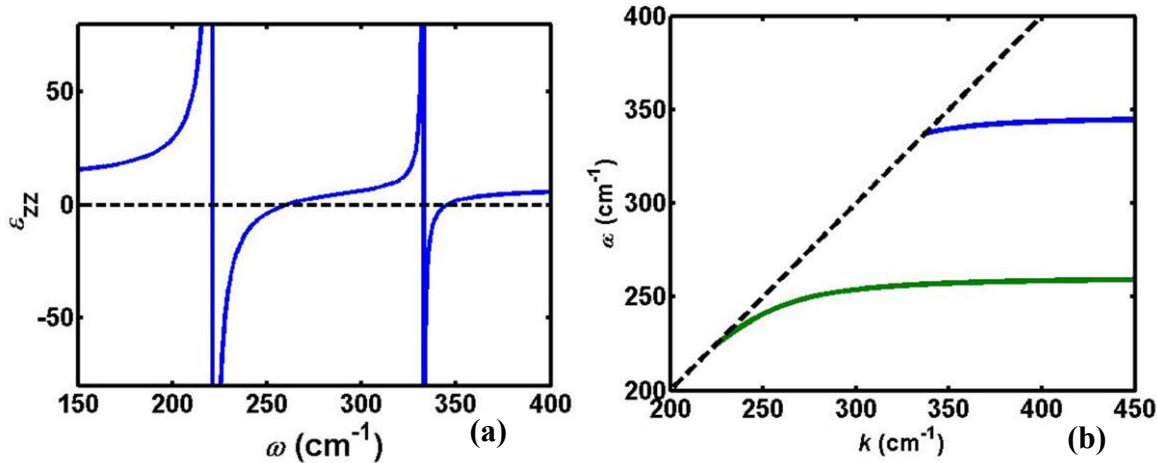
Since the attenuation constants of  $\beta_0$  and  $\beta$  was always positive, then the solution of surface dispersion relation in Eq.(8) existed if the value of effective permittivity was negative. Here, we defined the permittivity of each layer as

$$\varepsilon(\omega) = \varepsilon^\infty \left( \frac{\omega_{LO}^2 - \omega^2}{\omega_{TO}^2 - \omega^2} \right) \quad (9)$$

where  $\varepsilon^\infty$  represented dielectric constant for high frequency  $\varepsilon^\infty = 9.52$ ,  $\omega_{TO}$  was optical transverse frequency and  $\omega_{LO}$  described optical longitudinal frequency.

## Results and Discussion

In this study, we set the fractions of the two dielectrics with the same value,  $f=1$ . Here, we assumed permittivities for both dielectrics which compiled the bilayer system were isotropic. In our numerical calculation, we used parameters of InP in representing dielectric 1 with  $\omega_{TO} = 303.3 \text{ cm}^{-1}$ ,  $\omega_{LO} = 345.4 \text{ cm}^{-1}$  and  $\epsilon^\infty = 9.52$ . Here, we set randomly the parameters for dielectric 2 with  $\omega_{TO} = 220 \text{ cm}^{-1}$ ,  $\omega_{LO} = 250 \text{ cm}^{-1}$  and  $\epsilon^\infty = 7$ . The effective permittivity presented in Fig.2a.

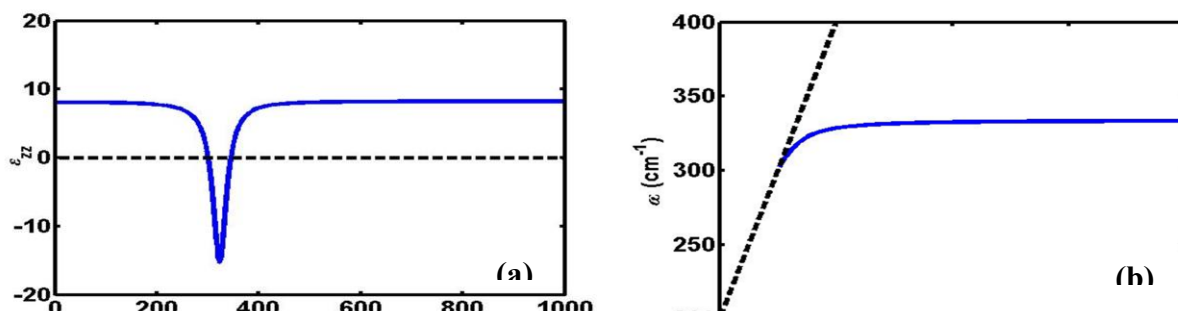


**Figure 2.** The effective permittivity  $\epsilon_{zz}^{eff}$  (a) and dispersion relation (b) of the bilayer system without intersection of the solutions of the each single dielectric.

In Fig.2a, it was shown that the effective permittivity had two resonant frequencies. One was contributed by dielectric 1, while the other was given by dielectric 2. Then there was two regions with negative permittivity. Hence, there were two solutions for surface polaritons for the bilayer system.

The results for the numerical solutions of dispersion relation in Eq.(8) was presented in Figure.(2b). Here, there was two branches of surface phonon polaritons. One branch located near  $\omega_{LO}$  of the first dielectric, while the other branch placed around  $\omega_{LO}$  of the second dielectric. The location of these branches agreed with the negative regions of the effective permittivity in Fig.(2a). The results in Fig.(2) showed that the effective medium behave simply as two dielectrics. The components of bilayer was not interfere each other. We thought that this happened because the resonant frequency of the second dielectric separated far enough from the resonant frequency of the first dielectric, as it can be seen from Fig.(2a).

In the second case, we used second dielectric with  $\omega_{TO} = 300 \text{ cm}^{-1}$ ,  $\omega_{LO} = 335 \text{ cm}^{-1}$ . Here, the result for effective permittivity was presented in Fig.(3a) while the solution of the dispersion relation was shown in Fig.(3b). It was shown that the effective permittivity of the bilayer system had only one resonant frequency near  $300 \text{ cm}^{-1}$  with also one region with the negative values.



**Figure 3.** The effective permittivity  $\epsilon_{zz}^{eff}$  (a) and dispersion relation (b) of the bilayer system with the intersection of the permittivity frequencies near the resonant frequencies of both dielectrics.

The surface dispersion relation was presented in Fig.(3b). There was only one branch of surface modes. This result was consistent with the effective permittivity in Fig.(3a). In our opinion, this result appeared because of the interference between first and second dielectrics. The interference was generated by the cross section of the frequency regions between  $\omega_{TO}$  and  $\omega_{LO}$  from the two dielectrics. The interference was also appeared because the difference of the optical frequency between the first and second dielectrics was very small.

### Conclusion

The appearance of the surface phonon polaritons in bilayer system was highly depend on the optical transverse and optical tangential frequencies of each dielectrics. We have two branches of dispersion relation curve when the optical frequencies from one dielectric separated far enough from the optical frequencies of the other dielectric. When there was optical frequency's cross section between those two dielectrics in bilayer system, we had only one branch of surface modes.

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